
CREATING THE SLOPE-INTERCEPT ($y = mx + b$) FORM OF A LINE

Some chapters back we played around with straight lines. We graphed a few, and we learned how to find their *intercepts* and *slopes*. Now we're ready to formalize the whole concept into a single idea — the kind of thing you'll need for your next algebra course, statistics, chemistry, business, economics, and many other disciplines.



Before we tackle these lines, let's review the algebra skills and straight-line concepts we'll need for this chapter.

Homework

1. Solve each formula for y :
 - a. $-4x + y - 1 = 0$
 - b. $-3x - y = 7$
 - c. $-2x + y + 1 = 0$
 - d. $3x + 2y = 10$
 - e. $2x - 7y = 14$
 - f. $-8x - 2y + 7 = 0$
2. Find all the intercepts of the line $3x - 7y = 42$.
3. Find the slope of the line connecting the points $(2, -3)$ and $(-8, 9)$.

□ CALCULATING THE SLOPE & Y-INTERCEPT OF A GIVEN LINE

Consider the line

$$y = 2x + 3$$

Let's see what we can discover about this line, using just the ideas from the previous chapters.

First, we'll determine the **slope** of the line. To do this, we need a pair of points on the line, which are simply solutions of the line equation.

To get points on the line, we'll choose a couple of x -values off the top of our head, and then calculate the corresponding y -values.

Suppose $x = 5$; then $y = 2(5) + 3 = 10 + 3 = 13$.

Thus, **(5, 13)** is a point on the line.

Now let $x = -1$; and so $y = 2(-1) + 3 = -2 + 3 = 1$.

Therefore, **(-1, 1)** is a point on the line.

We can now compute the slope using the points we just calculated, (5, 13) and (-1, 1):

$$m = \frac{\Delta y}{\Delta x} = \frac{13 - 1}{5 - (-1)} = \frac{13 - 1}{5 + 1} = \frac{12}{6} = 2$$

Second, we'll calculate the **y-intercept** of the line. Recall that the y -intercept of any graph is found by setting x to 0. Here's what we get:

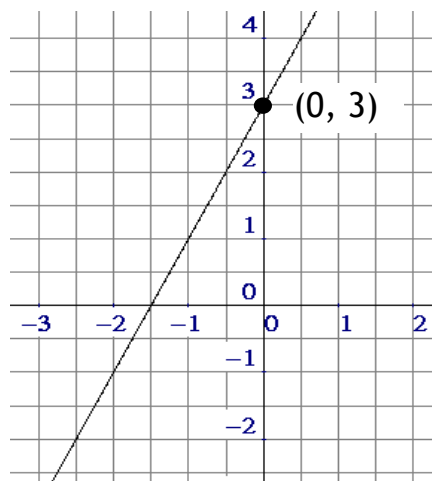
$$y = 2(0) + 3 = 0 + 3 = 3$$

The y -intercept is therefore **(0, 3)**. In summary,

The line

$$y = 2x + 3$$

has a slope of 2 and a y -intercept of (0, 3).



Homework

4. Consider the line $y = -3x + 7$.
- Find two points on the line. For instance, let $x = 1$ and then let $x = 3$ (or choose your own x 's).
 - Find the slope of the line using the two points you found in part a. by applying the definition of slope, $m = \frac{\Delta y}{\Delta x}$.
 - Find the y -intercept of the line (by letting $x = 0$, of course).
5. Consider the line $y = 5x - 14$.
- Find two points on the line. For instance, let $x = -2$ and then let $x = 4$.
 - Find the slope of the line using the two points you found in part a. by applying the definition of slope, $m = \frac{\Delta y}{\Delta x}$.
 - Find the y -intercept of the line by letting $x = 0$.
6. Consider the line $y = -x - 17$.
- Find two points on the line.
 - Find the slope of the line using the two points you found in part a.
 - Find the y -intercept of the line.
7. Consider the line $y = x + 44$.
- Find two points on the line.
 - Find the slope of the line using the two points you found in part a.
 - Find the y -intercept of the line.

□ THE $y = mx + b$ FORM OF A LINE

Here's a summary of the four homework problems you just completed (You did them, right?):

$y = -3x + 7$		$m = -3$ $y\text{-int} = (0, 7)$
$y = 5x - 14$		$m = 5$ $y\text{-int} = (0, -14)$
$y = -x - 17$		$m = -1$ $y\text{-int} = (0, -17)$
$y = x + 44$		$m = 1$ $y\text{-int} = (0, 44)$

See a pattern here? The slope of each line is simply the number in front of the x , the *coefficient* of the first term. For example, the slope of the line $y = 5x - 14$ is 5.



Important: Note that the slope of the line is **5**, NOT $5x$.

Also, the y -intercept is essentially the number hanging off the end of the equation. [“Essentially” means that the y -intercept is the *point* $(0, \text{something})$, and that *something* is the number at the end of the line equation.] For example, for the line $y = x + 44$, the y -intercept is $(0, 44)$. (Remember that some teachers and books will allow you to write the y -intercept simply as 44.)

Therefore, if you're asked for the slope and the y -intercept of the line $y = -14x + 17$, for example, you should be able to immediately reply that the slope is -14 and the y -intercept is $(0, 17)$.

Conversely, if you're asked to come up with the equation of the line whose slope is 23 and whose y -intercept is $(0, \pi)$, be sure you understand that no calculations are required to come up with the equation $y = 23x + \pi$.

Let's generalize the four examples above. We will, as usual, let m represent the slope of the line, and let b (for some odd reason) stand for the y -coordinate of the y -intercept. Here's what it all boils down to:

Creating the Slope-Intercept ($y = mx + b$) Form of a Line

$$y = mx + b$$



SLOPE



y-INTERCEPT

To be precise, b is not the y-intercept; b is the y-coordinate of the y-intercept. The y-intercept is properly written $(0, b)$.

EXAMPLE 1:

- A. Find the slope and the y-intercept of the line $y = -\frac{2}{3}x - 5$.

Answer: The slope is $-\frac{2}{3}$ and the y-intercept is $(0, -5)$.

- B. Find the equation of the line whose slope is -6 and whose y-intercept is $(0, \frac{4}{5})$. Answer: $y = -6x + \frac{4}{5}$.

EXAMPLE 2: Find the slope and the y-intercept of the line
 $3x - 5y + 15 = 0$.

Solution: The given equation, $3x - 5y + 15 = 0$, doesn't fit the slope-intercept form, $y = mx + b$, of a line. But we can make it fit; we can solve the equation $3x - 5y + 15 = 0$ for y :

$$\begin{array}{ll}
 3x - 5y + 15 = 0 & \text{(the original line)} \\
 \Rightarrow 3x - 5y = -15 & \text{(subtract 15 from each side)} \\
 \Rightarrow -5y = -3x - 15 & \text{(subtract 3x from each side)} \\
 \Rightarrow \frac{-5y}{-5} = \frac{-3x - 15}{-5} & \text{(divide each side by -5)}
 \end{array}$$

$$\Rightarrow y = \frac{3x}{5} + \frac{15}{5} \quad (\text{split the right-hand fraction})$$

$$\Rightarrow y = \frac{3}{5}x + 3 \quad (\text{rewrite the first fraction})$$

Now that the line is in the $y = mx + b$ form, we conclude that

The slope is $\frac{3}{5}$ and the y -intercept is $(0, 3)$.

Homework

8.
 - a. Find the slope and y -intercept of the line $y = -17x + 13$.
 - b. Find the equation of the line whose slope is 99 and whose y -intercept is $(0, -101)$
 - c. Find the equation of the line whose slope is $\frac{2}{3}$ and whose y -intercept is $(0, -\pi)$.
 - d. Find the equation of the line whose slope is $-\frac{5}{4}$ and whose y -intercept is $(0, 2\pi)$.

9. Find the slope and y -intercept of each line by converting the line to $y = mx + b$ form, if necessary:

a. $y = 132x - 1000$

b. $y = -\frac{8}{7}x - \frac{13}{9}$

c. $7x - 9y = 10$

d. $-3x - 5y + 1 = 0$

e. $2x + 7y = 13$

f. $-5x + 2y + 3 = 0$

g. $y = \frac{9x-5}{2}$

h. $-17x - y = 4$

i. $2x - 6y = 8$

j. $-x + 4y - 2 = 0$

k. $7y - 2x = 0$

l. $7x + 4y + 5 = 0$

□ A PROOF OF THE SLOPE—INTERCEPT FORM OF A LINE

We've learned the following:

The line with slope m and y -intercept $(0, b)$
has the equation $y = mx + b$.

We did this by viewing some homework results, but examples prove nothing. So let's do it the right way, by proving that $y = mx + b$ truly has the properties we've been claiming it has.

Claim #1: The line $y = mx + b$ has a y -intercept of $(0, b)$.

Proof: To find the y -intercept(s) of any graph, we set x to 0 and solve for y :

$$y = m(0) + b \Rightarrow y = 0 + b \Rightarrow y = b.$$

In other words, when $x = 0$, $y = b$, which means that $(0, b)$ is on the line, and thus $(0, b)$ is precisely the y -intercept.

Claim #2: The line $y = mx + b$ has a slope of m .

Proof: Our definition of slope, $m = \frac{\Delta y}{\Delta x}$, will be used to calculate the slope of the line $y = mx + b$. To apply this definition, we need two points on the line. One of them might as well be the y -intercept calculated above: $(0, b)$. For the other point, pick $x = 1$. This yields a y -value of $y = m(1) + b = m + b$. Therefore, $(1, m + b)$ is a second point on the line. Now we can find the slope, using the two points $(0, b)$ and $(1, m + b)$:

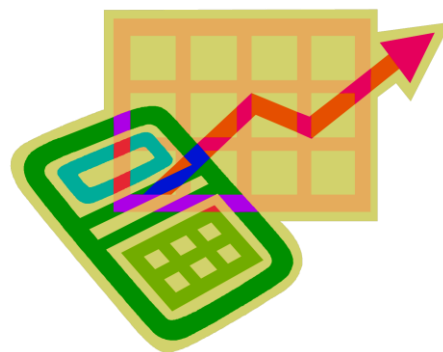
$$\text{slope} = \frac{\Delta y}{\Delta x} = \frac{(m + b) - b}{1 - 0} = \frac{m + b - b}{1} = \frac{m}{1} = m$$

and thus the slope of the line $y = mx + b$ is indeed m .

Our two claims have verified our assertion, and the proof is complete.

Review Problems

10. Consider the line $y = 10x - 13$.
- Find two points on the line. For instance, let $x = 1$ and then let $x = -2$.
 - Find the slope of the line using the two points you found in part a. by applying the definition of slope, $m = \frac{\Delta y}{\Delta x}$.
 - Find the y -intercept of the line by letting $x = 0$.
11. Find the equation of the line whose slope is -17 and whose y -intercept is $(0, 99)$.
12. Find the slope and the y -intercept of the line $-28x + 4y = 16$ by converting the line to $y = mx + b$ form.



Solutions

1. a. $y = 4x + 1$
- b. $-3x - y = 7$
- $$\Rightarrow -y = 3x + 7$$
- $$\Rightarrow \frac{-y}{-1} = \frac{3x + 7}{-1} \quad (\text{you could also multiply each side by } -1)$$
- $$\Rightarrow y = -3x - 7$$
- c. $y = 2x - 1$

$$\begin{aligned}
 \text{d. } 3x + 2y &= 10 &\Rightarrow 2y &= -3x + 10 \\
 &&\Rightarrow y &= \frac{-3x + 10}{2} \\
 &&\Rightarrow y &= -\frac{3}{2}x + 5
 \end{aligned}$$

$$\text{e. } y = \frac{2}{7}x - 2$$

$$\text{f. } y = -4x + \frac{7}{2}$$

$$2. \quad x\text{-int: } (14, 0); \quad y\text{-int: } (0, -6)$$

$$3. \quad m = -\frac{6}{5}$$

$$4. \quad \text{a. } (1, 4) \text{ and } (3, -2) \qquad \text{b. } m = \frac{\Delta y}{\Delta x} = \frac{4 - (-2)}{1 - 3} = \frac{6}{-2} = -3$$

$$\text{c. } y = -3(0) + 7 = 7; \text{ so the } y\text{-intercept is } (0, 7).$$

$$5. \quad \text{a. } (-2, -24) \text{ and } (4, 6) \qquad \text{b. } m = \frac{\Delta y}{\Delta x} = \frac{-24 - 6}{-2 - 4} = \frac{-30}{-6} = 5$$

$$\text{c. } y = 5(0) - 14 = -14; \text{ so the } y\text{-intercept is } (0, -14).$$

$$6. \quad \text{a. } \text{You choose the two points.} \quad \text{b. } m = -1 \qquad \text{c. } y\text{-int} = (0, -17)$$

$$7. \quad \text{a. } \text{You choose the two points.} \quad \text{b. } m = 1 \qquad \text{c. } y\text{-int} = (0, 44)$$

$$8. \quad \text{a. } m = -17 \quad y\text{-int} = (0, 13) \quad \text{b. } y = 99x - 101$$

$$\text{c. } y = \frac{2}{3}x - \pi \qquad \text{d. } y = -\frac{5}{4}x + 2\pi$$

$$9. \quad \text{a. } m = 132 \quad y\text{-int} = (0, -1000) \quad \text{b. } m = -\frac{8}{7} \quad y\text{-int} = (0, -\frac{13}{9})$$

$$\text{c. } m = \frac{7}{9} \quad y\text{-int} = (0, -\frac{10}{9}) \quad \text{d. } m = -\frac{3}{5} \quad y\text{-int} = (0, \frac{1}{5})$$

$$\text{e. } m = -\frac{2}{7} \quad y\text{-int} = (0, \frac{13}{7}) \quad \text{f. } m = \frac{5}{2} \quad y\text{-int} = (0, -\frac{3}{2})$$

$$\text{g. } m = \frac{9}{2} \quad y\text{-int} = (0, -\frac{5}{2}) \quad \text{h. } m = -17 \quad y\text{-int} = (0, -4)$$

$$\text{i. } m = \frac{1}{3} \quad y\text{-int} = (0, -\frac{4}{3}) \quad \text{j. } m = \frac{1}{4} \quad y\text{-int} = (0, \frac{1}{2})$$

$$\text{k. } m = \frac{2}{7} \quad y\text{-int} = (0, 0) \quad \text{l. } m = -\frac{7}{4} \quad y\text{-int} = (0, -\frac{5}{4})$$

10. a. $(1, -3)$ and $(-2, -33)$

b. $m = \frac{\Delta y}{\Delta x} = \frac{-3 - (-33)}{1 - (-2)} = \frac{-3 + 33}{1 + 2} = \frac{30}{3} = 10$

c. $y = 10(0) - 13 = 0 - 13 = -13$; the y -intercept is $(0, -13)$.

11. $y = -17x + 99$

12. $m = 7$; y -int: $(0, 4)$

□ *TO ∞ AND BEYOND*

Consider the infinite sequence of numbers:

8, 10, 12, 14, 16, . . .

If 8 is the 1st term, and 10 is the 2nd term, etc., what is the 1,000th term?

“Without education, we are in a horrible and deadly danger of taking educated people seriously.”

– G.K. Chesterton